Uncertainty and the de Finetti tables[1]

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The new paradigm in the psychology of reasoning adopts a Bayesian, or probabilistic, model for studying human reasoning. Contrary to the traditional binary approach based on truth functional logic, with its binary values of truth and falsity, a third value that represents uncertainty can be introduced in the new paradigm. A variety of three-valued truth table systems are available in the formal literature, including one proposed by de Finetti. We examine the descriptive adequacy of these systems for natural language indicative conditionals and bets on conditionals. Within our framework the so-called "defective" truth table, in which participants choose a third value when the antecedent of the indicative conditional is false, becomes a coherent response. We show that only de Finetti's system has a good descriptive fit when uncertainty is the third value.

Keywords: New paradigm psychology of reasoning; Indicative conditionals; Bets on Conditionals; Uncertainty and three-valued tables; de Finetti tables.

The new Bayesian, or probabilistic, paradigm in the psychology of reasoning (Oaksford & Chater, 2007, 2009; Over, 2009; Pfeifer & Kleiter, 2010) implies a close parallel relationship between assertions of the indicative conditional of natural language, *if A then C*, and other uses of conditionals, particularly bets on conditionals, *I bet that if A then B*. Politzer, Over, and Baratgin (2010) explain how this predicted relation goes back to Ramsey (1926/1990, 1929/1990) and de Finetti (1936/1995, 1937/1964) and provide experimental evidence that this parallel relation exists (see also Baratgin, Over, & Politzer, in press).

There are two aspects to the parallel relation. First, the subjective probability of the conditional assertion is the subjective conditional probability of *C* given *A*, *P(if A then C)* = *P(C|A)*, and the subjective probability that the conditional bet will be won is also *P(C|A)*. Second, the conditional assertion will be true, and the conditional bet will be won, when *A* and *C* are both true, and the conditional assertion will be false, and the conditional bet lost, when *A* is true and *C* is false. When *A* turns out to be false, no indicative assertion or bet is made. The assertion and bet are *void*: the indicative conditional *if A then C* is neither true nor false, and the bet on it is neither won nor lost. The finding that the indicative conditional has such a three-valued table goes back to Wason (1966), and psychologists long called it the *defective truth table*, as if there were something wrong with the table or the participants for producing it (see Evans & Over, 2004, for a review). Psychologists who used "defective" in a negative sense were apparently unaware that this table is found in the formal systems of a number of philosophers, logicians, linguists, and mathematicians, including de Finetti (1936/1995), Cooper (1968), Jeffrey (1963), Farrell (1979), Calabrese (1987), and Goodman, Nguyen, and Walker (1991) (see Baratgin, Politzer, & Over, under review, for a survey of these formal conditional tables). A conditional *if A then C* for which *P(if A then C)* = *P(C|A)* has been called a *probability conditional* (Adams, 1998; Oaksford & Chater, 2007, 2009) and a *conditional event* (de Finetti, 1936/1995; Pfeifer & Kleiter, 2010). We will prefer the term "conditional event" when referring to the three-valued "defective" table, which should anyway be called the 2x2 *de Finetti table* for this conditional as he was the first to construct it in a major theory of the conditional. See Table 1.

Insert Table 1 about here.

The identity, $P(\textit{if } A \textit{ then } C) = P(C|A)$, has such deep consequences for the study of conditional reasoning that it has been called *the Equation* by both philosophers (Edgington, 1995) and psychologists (Oaksford & Chater, 2007, 2009). Psychologists have provided strong support for the Equation as a description of people's actual probability judgments about conditionals (Douven & Verbrugge, 2010; Evans, Handley, Neilens, & Over, 2007; Evans, Handley, & Over, 2003; Fugard, Pfeifer, Mayerhofer, & Kleiter, 2011; Oberauer & Wilhelm, 2003; Over, Hadjichristidis, Evans, Handley, & Sloman, 2007; Politzer et al., 2010). The most common account of how people make the judgment, $P(\textit{if } A \textit{ then } C) = P(C|A)$, is that they use the *Ramsey test* (Edgington, 1995; Ramsey, 1929/1990). They hypothetically suppose *A*, making whatever mental changes are necessary to preserve consistency, and judge to what extent *C* follows. Some more specific proposals have been made about how people implement the Ramsey test, as in the suppositional account of Evans and Over (2004), but we do not need to take a stand on these hypotheses in this paper.

Our aim here is to extend our study of *if A then C* to a context in which its components, *A* and *C*, are uncertain (Politzer et al., 2010; Baratgin et al., in press). Previous studies of the three-valued, "defective" table, going back to Wason (1966), have had the limitation that they were conducted under conditions of certainty. Consider the conditional studied in Politzer et al. (2010):

(1) If the chip is square (*S*) then it is black (*B*).

A distribution of chips was displayed for the participants in the study. There were two black round chips, one white round chip, three black square chips, and one white square chip. The participants were told that “the chip” referred to the chip that was going to be randomly selected from this distribution, and they were also asked to consider the following bet on the conditional:

(2) I bet that if the chip is square (*S*) then it is black (*B*).

Politzer et al. followed the standard, and indeed universal, practice of presenting what were in effect the four cells of 2x2 truth table - *S* true and *B* true, *S* true and *B* false, *S* false and *B* true, and *S* false and *B* false - in conditions of certainty. The participants were to say whether, in each of these cells, the conditional (1) was true, false, or neither true nor false, and whether the conditional bet (2) was won, lost, or neither won nor lost. In similar truth table tasks, cards have been used with letters and numbers, or colours and shapes, and a conditional was given about these items (Evans & Over, 2004). In all these tasks, the participants were asked to suppose that the four cells successively held, or were successively shown items exemplifying these cells, and asked for their judgments about the conditional. Under these conditions of certainty, the responses were generally consistent with the three-valued, “defective” table, adding to the evidence of a close relation between indicative conditionals and bets on conditional bets (see also Oberauer & Wilhelm, 2003).

The new paradigm, however, stresses that most judgment and reasoning in everyday life and in science takes place in a context of at least some uncertainty (Oaksford & Chater, 2007, 2009). For example, the antecedent and consequent of the following indicative conditional are uncertain for most people:

(3) If global warming continues (*W*) then London will be flooded (*L*).

We could, following the traditional paradigm of studying truth table tasks under certainty, ask people for their judgments about (3) after asking them simply to assume that both *W* and *L* are true, that *W* is true and *L* is false, and so on. But what are their judgments given that both *W* and *L* are to a degree uncertain? Such questions about uncertainty have not yet been addressed in psychological research. The only previous study of extended truth tables in the psychological literature is Elqayam (2006), which is concerned with the important but distinct topic of the semantic paradoxes.

The question of participants' judgments in such a 3x3 task was even not raised in the traditional paradigm, and apparently no predictions follow for it in this paradigm (Evans & Over, 2004, report no work at all on 3x3 truth table tasks in their review). However, the new paradigm, with its emphasis on uncertainty, does imply what the responses should be in the 3x3 task, in an extended table worked out by de Finetti himself. Thus we can distinguish between the 2x2 de Finetti table, see Table 1 again, and the 3x3 de Finetti table, see Table 2, for the conditional event. In fact, de Finetti (1936/1995) also proposed three-valued tables for negation, *not-A*, conjunction, *A & C*, disjunction, *A or C*, and the material conditional, equivalent to *not-A or C*, when *A* and *C* can be true, false, or uncertain (Baratgin et al., under review). These tables can be called de Finetti tables for the connectives, but the study we report here focuses on whether participants' judgments about the natural language indicative conditional and the bet on the conditional conform to the 3x3 de Finetti table for what he called the conditional event or to some other formal table found in the philosophical, logical, linguistic, and mathematical literatures.

Insert Table 2 about here

Before going on to our experimental work, let us clarify the notion of uncertainty that we are focusing on and introduce some terminology. To make the example more concrete, let us suppose that the assertion of (3) is understood in context to be about the next ten years. Suppose it turns out to be definitely false that global warming has continued, and so we judge that $P(W) = 0$. Then the indicative conditional (3) is void, because there is no indicative fact to settle the matter, and we would switch to a counterfactual conditional, *If global warming had continued, then London would have been flooded*. Even though $P(W) = 0$, we can still say that the probability of this counterfactual is the conditional probability, $P(L|W)$, with this value being high or low depending on a Ramsey test for the counterfactual (Gilio & Over, 2012; Over et al., 2007). In effect, the uncertainty of the indicative conditional is transferred to the counterfactual when $P(W) = 0$. (See Pfeifer, this issue, and Pfeifer & Kleiter, 2010, for more on conditionals with antecedents having 0 probability.)

In a 2x2 table generally for *if A then C*, where *A* and *C* are only classified as true or false, the four cells are often labelled with letters. The *a* cell is where *A* and *C* are true, the *b* cell where *A* is true and *C* is false, the *c* cell where *A* is false and *C* is true, and the *d* cell where *A* is false and *C* is false. In the 3x3 table, there are five extra cells. Consider the extra cell between the *a* and *b* cells, in which *A* is true but *C* is uncertain. We will call this the *a/b* cell, because the *a* cell or alternatively the *b* cell will result once the uncertainty about *C* is resolved, i.e. *C* is found to be certainly true or certainly false. There is also clearly the *c/d* cell, the *a/c* cell, and the *b/d* cell. At the centre of the table, there is the final extra cell, the *a/b/c/d* cell, when both *A* and *C* are uncertain. See Table 3 for all these cells.

Insert Table 3 about here.

There has been no psychological research on Table 3 when the third value is uncertainty as we are using the term (but see again Elqayam, 2006), and more specifically on the five important uncertain states of mind corresponding to the *a/b, c/d*, *a/c*, *b/d* and *a/b/c/d* cells. In the psychological literature, the role of uncertainty in premises of deductive arguments has been studied (see for a review Politzer, 2007). Some studies have outlined how far participants endorse a conclusion derived from uncertain premises for *Modus Ponens* and *Modus Tollens* (George, 1995, 1997, 1999; Politzer & Bourmaud, 2002; Stevenson & Over, 1995). The main result is that the rate of endorsement is linked to the degree of uncertainty related to the premises. Participants attach a degree of uncertainty to the conclusion when at least one premise has a degree of uncertainty, and also that there is a relationship between the degree of uncertainty in the premises and the degree of uncertainty for the conclusion of both *Modus Ponens*, and *Modus Tollens*. However, no experiment conducted directly on Table 3 has investigated how the evaluation of a conditional as a whole - as true, false, or uncertain - depends on the uncertainty of its antecedent and consequent. For a full account of people's understanding of conditionals and uncertainty, it is necessary to discover how they respond in the five new cells. This is the main ambition of our study.

Following de Finetti (1936/1995), we can also think of the de Finetti conditional event table as a description of the outcomes of a bet on a conditional. Let us say that we are arguing with other people about climate change, asserting (3) and then offering them the bet in (4):

(4) We bet that, if global warming continues (*W*), then London will be flooded (*L*).

Keeping the context about the next ten years, we win the argument to support our assertion of (3), and the bet on the conditional (4) we make as part of it, when the time passes and we observe that global warming continued and London has flooded, the *a* cell; and we lose both when we observe that global warming has continued but London has not flooded, the *b* cell. During much of the ten year period, we may well be uncertain whether global warming will continue and London will flood, and then we cannot yet settle the argument or the bet. But in the *c*, *c/d* and *d* cells, when global warming stops, we do not win or lose the argument or the bet about the indicative conditional, for there is then no indicative fact that makes (3) true or false and that settles the argument and the bet. Although the indicative (3) and the bet on it (4) are now void, there is still uncertainty about what would have happened had global warming continued, about the counterfactual in effect, and the *c*, *c/d* and *d* cells can be thought of as having uncertainty in this sense. In this paper, we will look for participants' evaluations in the five newly introduced cells, and examine whether the close parallel relation between assertions of conditionals and bets on conditionals extends to these cells. We will use novel experimental materials and generalise the results of Politzer et al. (2010) to all nine cells of the 3x3 table.

METHOD

Materials

The stimuli were individually presented on a computer screen. Participants saw a computer representation of a “game” composed of two opaque boxes, one on top of the other, with some open and clear space between them. The top box had square or round chips in it that were black or white. A random chip would sometimes fall from the top box to the lower box, and then a photo would be taken of this chip while it passed through the space between the boxes. The photos were sometimes taken through filters that had the effect of concealing the shape or the colour of the chips, or both the shape and the colour (see Figure 1).

Insert Figure 1 about here

Consequently the shape of the chips the participants could see had three possible values: the chips could appear as square, round, or blurred. The colour of the chips the participants could see could also take three values: black, white, or grey. There were overall nine possibilities (3x3 combinations) corresponding to the nine cells of Table 3. See Figure 2.

Insert Figure 2 about here

An elementary scenario described two children playing a game with this apparatus. A child, Marie, selected a chip at random and let it fall from the top, and another child, Pierre, had to guess which kind of chip (the shape and colour) this was. He could say, for instance, *If the chip is square, then it is black,* or *I bet that if the chip is square, then it is black*, so reproducing the two conditions - indicative conditional (IC) and bet on conditional (BC) - studied by Politzer et al. (2010). The task for the participants was to decide, for each of the nine “photographs”, whether what Pierre said was true, false, or neither (for IC), or whether the bet was won, lost, or neither (for BC). The option “neither” here covers the senses of uncertainty that we discussed above.

Insert Figure 3 about here

Other connectives were also studied (e. g., conjunction, *The chip is square and black*), and the simple affirmation, *The chip is black*. The latter was always presented first and was used to familiarise participants with the task.

Participants

A total of 192 French native speakers, students (remote teaching) at the University Paris 8 volunteered for the experiment. Their mean age was 37. They already held a degree and resumed their studies (social sciences). They had no specific background in logic or probability theory.

Design and procedure

The participants were randomly allocated to the two conditions: the Indicative Conditional (IC) or the Bet on conditional (BC), using the same indicative conditional, *if the chip is square, then it is black.* They were required to answer nine randomly ordered questions (see Figure 2), each question corresponding to a cell of the table. The question was presented in a 3-option multiple choice format, to which they answered by selecting one option (Figure 4).

Insert Figure 4 about here

RESULTS

Data analysis

For the simple affirmation, all participants correctly answered. This is evidence that they recognized and understood the iconic representation of the three possibilities for the antecedent, that is, the square, "blurred", and round shapes were visually well distinguished, with the "blurred" icon representing uncertainty about the shape; and similarly for the consequent: the black, grey, and white colours were visually well distinguished, with the grey shade representing uncertainty about the colour.

For the conditional questions, the tables produced by participants will be analysed in two stages.

The first stage casts the results in terms of the traditional classification. We categorize the answers in the four "old" cells of the traditional table that correspond to the four cases where the antecedent and consequent are either true or false (*a*, *b*, *c*, *d* cells of Table 3). The results are presented in Table 4.

Insert Table 4 about here

The patterns of response for conditions IC and BC were nearly identical ($X^2 = 3.92$, df = 4, p = .41) and reproduce traditional observations (at least qualitatively) in the literature (for a review see Over & Evans, 2004). The table traditionally called "defective", Table 1, was participants' modal interpretation in both conditions (41.7%). The second most frequent response was the conjunction table (34.9%). A much smaller minority (13%) of the participants displayed the material conditional table, and a still smaller minority the material bi-conditional table and the bi-conditional table, which represents the bi-conditional event in de Finetti's terms.

In the second stage of the analysis, we further characterize the tables by considering all nine cells. The observed three-valued tables are compared with the relevant three-valued formal tables in the logical, linguistic, philosophical, and mathematical literatures. This analysis now focuses on the five "new" cells of the table (the *c/d*, *a/c*, *b/d, a/c*, and *a/b/c/d* cells of Table 3). Each participant's observed table is classified by considering the formal table to which it is the *closest*. Our criterion of "closeness" or "distance" is as follows. A participant's table is taken to be a perfect instance of a formal table N when it is identical to N. A participant's table is a close instance of N when it differs from N just by one cell (and from any other formal table by more than one cell). If a participant's table differs equally (by one cell) from two (or more) formal tables, it is close but classified as *ambiguous* between these tables (these are equally likely). Finally, if a participant's table differs by two or more cells from all formal tables, then it is classified as *indeterminate* as it differs too much to make a reliable identification. The results are presented in Table 5.

Insert Table 5 about here

Almost all of the participants' tables coincide with a system of three-valued tables proposed in the formal literature.

The patterns in the two conditions, IC and BC, are parallel and almost identical for the three main types of interpretation of the conditional: conditional event, conjunction, and material conditional (all chi-square values yield $p > .36$, $df=4$).

For each of the three major interpretations observed - conditional event, conjunction, and material conditional - the majority of the observations coincides with a table coherent with de Finetti's three-valued logical system (de Finetti, 1936/1995). In other words, more than half of the conditional event interpretations coincide with de Finetti's specific conditional event (while the remaining conditional event interpretations are distributed over other conditional event tables in the formal literature, such as Farrell's, and Cooper's) and the same obtains for the conjunctive and material conditional interpretations of the conditional).[1]

For both conditions IC and BC pooled together, 57.7% of the observations identified as conditional event tables coincided with de Finetti's specific conditional event table and approximately 20% were a Farrell conditional table (Farrell, 1979; Goodman, Nguyen, and Walker, 1991), which differs from de Finetti's table only for the *b/d* cell (see Table 5). The two other types of table with a frequency exceeding 10% coincide with the Cooper conditional event table (Cooper, 1968, Calabrese; 1987), in which the *a/c* and *b/d* cells differ from de Finetti's table, and an ambiguous table which differs only by one cell from de Finetti's table (*a/c* cell) and Cooper's table (*b/d* cell).

De Finetti's conjunction table was chosen by 52.2% of the participants who had a conjunctive interpretation (averaged across both conditions). Porte's conjunction table (Porte,

1958) was the second most frequently observed table (28.4% of the participants). This table differs from de Finetti's conjunction table in the *b/d* cell. In addition, there were many de Finetti conjunction tables in the category "ambiguous conjunction table" (that is, one out of the two candidate tables was a de Finetti table).

In the IC condition, 43% of the participants who had a material conditional response produced de Finetti's material conditional table. The McCathy material conditional table (McCarthy, 1963) represents 35% of participants' answers out of those who gave a material conditional response. McCathy's table only differs from de Finetti's in the *a/c* cell (see Table 5). In the BC condition, de Finetti's material conditional table predominates (72.7%) out of those giving a material conditional interpretation.

Among the very few participants who had a material bi-conditional interpretation and a bi-conditional interpretation, de Finetti's material bi-conditional table and de Finetti's bi-conditional (event) table were the majority answer, respectively.

DISCUSSION

In this experiment, we have categorized participants' response tables by analysing how close they are to relevant tables in the formal literature (see Baratgin et al., under review, for more on this). We wish to single out three results here as worthy of special note.

The first result is that there are a limited number of tables that actually account for people's responses. The 2x2 "defective" truth table can be theoretically extended with a third value to a 3x3 table in 243 (3^5) ways. However, the responses of our participants are by no means scattered among all these possibilities. Our evidence is that people respond to uncertainty in a limited number of ways, and these are found in the formal literature (Baratgin et al., under review).

The second main result is that a majority of participants conform to the 3x3 de Finetti conditional event table in both the indicative conditional , IC, and the bet on conditional, BC, conditions, and in the same proportion. This result reinforces the new paradigm argument that participants are not "defective" but coherent. The finding that there is not a difference between the two conditions generalizes Politzer et al. (2010). People treat questions about the truth or the falsity of the assertion of indicative conditionals as very similar to questions about winning or losing bets on conditionals.

The third main result is that there is a minority response conforming to the conjunctive table in both the IC and BC conditions. A minority conjunctive response has been found in experiments on participants' judgments about the probability of conditionals like (1), but not (or not so much) like (3) that are more realistic or familiar (compare Evans et al., 2003, with Over et al., 2007). There is evidence that this response is due to a processing limitation (Politzer et al., 2010; Evans et al., 2007). But another possible explanation for what we have found here is the "matching bias" of Evans (1972). People tend to use "true" for the cases that match items in the conditional statement and "false" for those that do not. Consider (1) again, *If the chip is square, then it is black*. There is a match with the photo of a square and black chip in the *a* cell, which makes the conjunction *The chip is square and black true*. The *b*, *b/d*, *c*, *c/d*, and *d* cells are mismatching instances and make the conjunction false. It is visually uncertain whether there is a full match in the remaining cells, the *a/b*, *a/c*, and *a/b/b/d* cells, and these are uncertain cases in de Finetti's table for conjunction.

We can make further points about our experiment and how to develop it by recalling the traditional task in a context of uncertainty, the *Wason selection task* (Evans & Over, 2004; Wason, 1966). Imagine a Wason selection task based on (1). There are four cards describing a chip. One side of a card says whether the chip is square or round and the other side whether it is black or white. The cards are lying on a table with only one side revealed. The *S card* states that a square chip has been selected, with the hidden side indicating whether it is black or white. The *not-S card* states that a round chip has been selected, with the hidden side indicating whether it is black or white. The *B card* states that the chip is black, but it is hidden whether it is square or round. And the *not-B card* that the chip is white, with it being hidden whether it is square or round. The question that the participants are asked is which cards should be turned over to decide whether or not (1) holds.

There is a clear parallel relationship between such a selection task and our experiment. The *not-S* card corresponds to the c/d cell of Table 3. It makes the indicative (1) void by the de Finetti table, and there is no need to turn it over. The related counterfactual*, If the chip had been square then it would have been black*, is completely uncertain, unless further information is given on the square chips, such as that all, none, or some proportion of square chips are black (see Evans & Over, 2004, on counterfactuals). The other cards keep (1) uncertain as a result of a lack of visual information, and there are clearly good reasons to turn these over given the de Finetti table. The *S* card corresponds to the a/b cell, the *B* card to the a/c cell, and the *not-B* card to the b/d cell. Finally, the a/b/c/d cell in the middle of Table 3, where both *S* and *B* are uncertain, would be like a card that is not seen at all and is not used in a traditional selection task.

The traditional, bivalent claim about the selection task was that only the *S* and *not-B* cards should be turned over to test whether (1) is true or false (Wason, 1966). This claim, however, came under serious criticism in some of the first work on the new paradigm (Evans & Over, 1996, 2004; Oaksford & Chater, 1994). The traditional presupposition is that conditionals like (1) are material conditionals, logically equivalent to *not-S or B*, making (1) true when the *not-S* card, the c/d cell, is shown or the *B* card, the a/c cell, is shown. In contrast, de Finetti's conditional event - the probability or suppositional conditional - of the new paradigm is uncertain for these cards and cells. We have confirmed the existence of this uncertainty in our study here of 3x3 tables. From a new paradigm point of view, the *S*, *B*, and *not-B* cards should all be turned over, though the *S* card is the most decisive since the *B* and *not-B* cards could result in a void outcome. The most common choice in a selection task is the *S* card followed by the *B* and *not-B* cards (Evans & Over, 2004). One way to develop this research would be to construct a selection task for a bet like (2), using the same cards but with the question becoming which cards should be turned over to determine whether the bet is won or lost. The new paradigm again implies that the *S*, *B*, and *not-B* cards should all be turned over. But an additional aspect of a betting-based selection task would be the expected utility of turning over a given card, depending on how much money was bet, and this aspect could also be described in a new paradigm, Bayesian approach (see Evans & Over, 2004, and Oaksford & Chater, 1994, on deontic selection tasks).

Our materials are close to the way in which de Finetti (1936/1995,1937/1964) described his table. Some of what de Finetti writes on his tables is open to more than one interpretation, but he is clear that the uncertainty value is "superimposed" on the underlying classification of *A* and *B* as either true or false (de Finetti, 1936/1995). As part of his account of subjective probability, the entries in his tables are epistemic states. People can definitely know, or be certain, that *A* or that *B* are true or false, or be uncertain to some extent of *A* or of *B*. Our participants were in these epistemic states. They could see clearly the shape or colour of a chip, or they could be uncertain of the shape or of the colour. The third value is not, for de Finetti, on the same "level" as truth and falsity and standing between the two - he compared the uncertainty value in his tables to an entry of "sex unknown" in a statistical survey (de Finetti, 1936/1995). In this respect, he is radically different from some other logicians, who have viewed the third value as non-epistemic and objective in some sense.

Łukasiewicz (1970), for example, appears to have thought of the future as undetermined in a non-epistemic sense. In his view, it is apparently not simply an epistemic limitation that prevents us from knowing whether it is true or false that London will flood in the next ten years (see Haack, 1974, on his three-valued system, and compare her on Kleene, 1952, who seems to have had a view closer to de Finetti's). We did not, of course, investigate evaluations of future possibilities. In our materials, a chip has a definite shape and colour whether or not these properties are obscured by a filter. The filter introduces epistemic uncertainty into our experiment, and that makes the table of our participants' responses of the same epistemic type as that proposed by de Finetti.

Note also that the subjective value of uncertainty might be progressively divided into finer and finer epistemic distinctions, indicating more or less uncertainty, with the three-valued table becoming four-valued and so on (and interval-valued probability judgments would naturally go with these uncertainty judgments; see Pfeifer & Kleiter, 2010). Our study could be developed by making the random chip more or less likely to be square or round and black or white, by having it selected from a frequency distribution. Eventually, the "uncertain" value for *if S then B* could be replaced by the conditional probability *P(B|S)* itself (Edgington, 1995; Jeffrey, 1991). The resulting table would go with de Finetti's full many-valued logic of probability (Finetti (1936/1995), and some anomalies of the restricted three-value system could be avoided. For instance, in the three-valued system, a logically true conditional like *if S & B then B* will be uncertain by the 3x3 de Finetti table supposing *S & B* is uncertain, but of course *P(B|S & B)* = 1, and *if S & B then B* is never intuitively uncertain. Such oddities do not arise supposing the value of *if S & B then B* is *P(B|S & B)* when *S & B* is uncertain.

Distinguishing different degrees of uncertainty and probability in tables should lead to an investigation of neglected topics in the psychology of reasoning. For example, people will sometimes make *abductive* inferences (Peirce, 1931-1958) to a degree of confidence in *A* as an *explanation* of *C* when they have some degrees of confidence in *C* and *if A then C*.

Observing that London is flooded, people might increase their confidence that global warming has continued, because they have high confidence that this explains the flood. The old paradigm generally dismissed this form of reasoning as the fallacy of affirming the consequent, but it can sometimes be a justified inference according to the new paradigm, especially when it is part of dynamic Bayesian belief revision (Oaksford & Chater, 2007, this issue). Abduction can be studied in experiments in which a marginal degree of confidence in *A* in a table is inferred from confidence in *if A then C* and *C*.

Research will also have to be done on the psychological representation and processing of uncertainty. There are some new paradigm hypotheses that radically extend old paradigm mental model and mental logic theories to epistemic or probabilistic accounts (Evans, 2007; Evans & Over, 2004; Oaksford & Chater, 2010; Pfeifer & Kleiter, 2010). However, these hypotheses will have to be tested by going far beyond the limitations of traditional studies of the "defective" truth table in the old paradigm to experiments on people's judgments under uncertainty.

Note

[1] For each of the three interpretations, the table ranked second (Farrell's for the conditional event, Porte's for the conjunction, and McCarty's for the material conditional) differs from de Finetti's tables by just one cell. If these responses are regarded as de Finetti's tables with one error, de Finetti's tables exceed 75% of the responses.

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Figure 1. The game presentation, with the different possible photographs

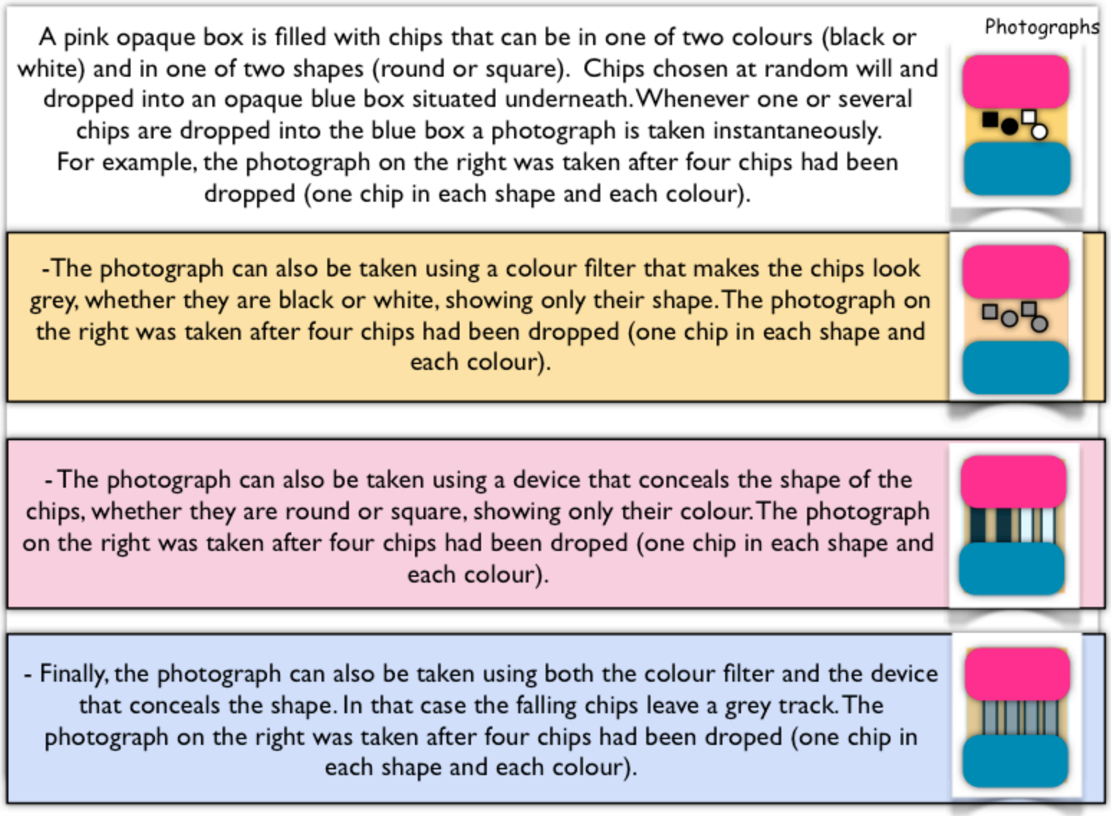

Figure 2: The nine possible combinations of photographs correspond to the nine cell of the three-valued truth table. The target sentence was: *If the chip is square, then it is black.*

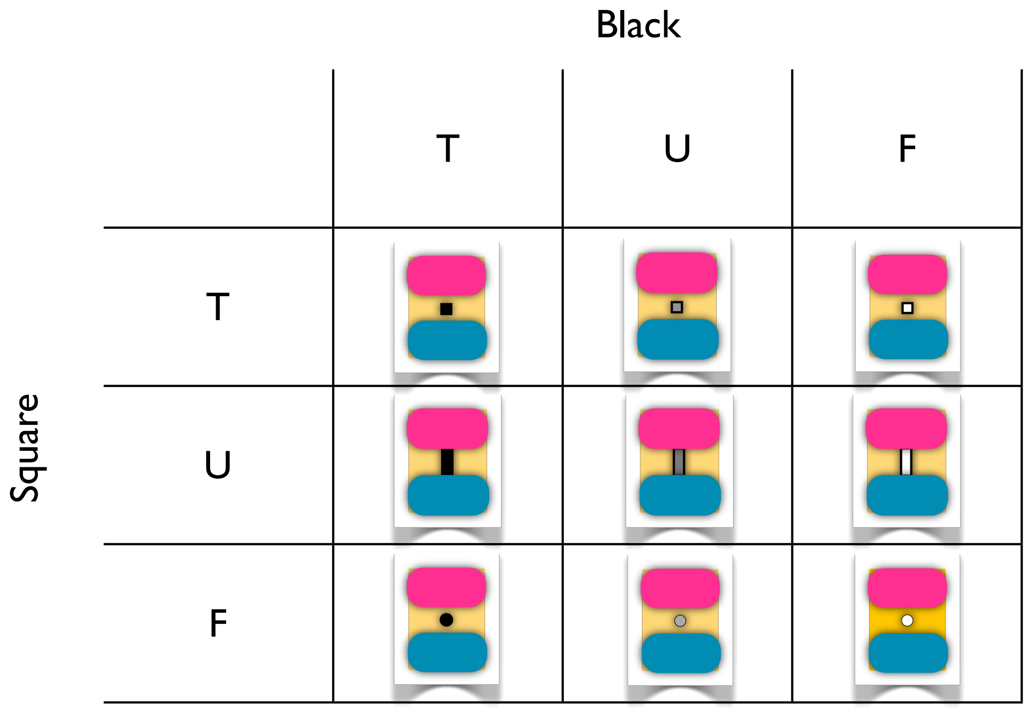

Figure 3. The elementary scenario and the conditional sentences in the indicative conditional and the betting conditions.

Two children, Pierre and Marie are playing to make chips fall.
Marie chooses chips at random and drops them one by one, out of Pierre's sight.
Pierre has to guess the shape and colour of every chip.

Marie chooses one chip at random and drops it.
Pierre who did seen the chip being drop says:

Indicative conditional condition (IC)

"If the chip is square, then it is black"

Pierre's statement may be true or false. For you to decide, you will have at your disposal photographs taken as just explained.

For each of the following photographs, you will indicate whether Pierre's statement is true or false by selecting the option that coincides with your response.

Conditional Bet condition (BC)

"I bet that if the chip is square, then it is black"

Pierre's bet may be won or lost. For you to decide, you will have at your disposal photographs taken as just explained.

For each of the following photographs, you will indicate whether Pierre's bet is won or lost by selecting the option that coincides with your response.

Figure 4. Example for the situation in which the antecedent is uncertain and the consequent false for IC and BC conditions.

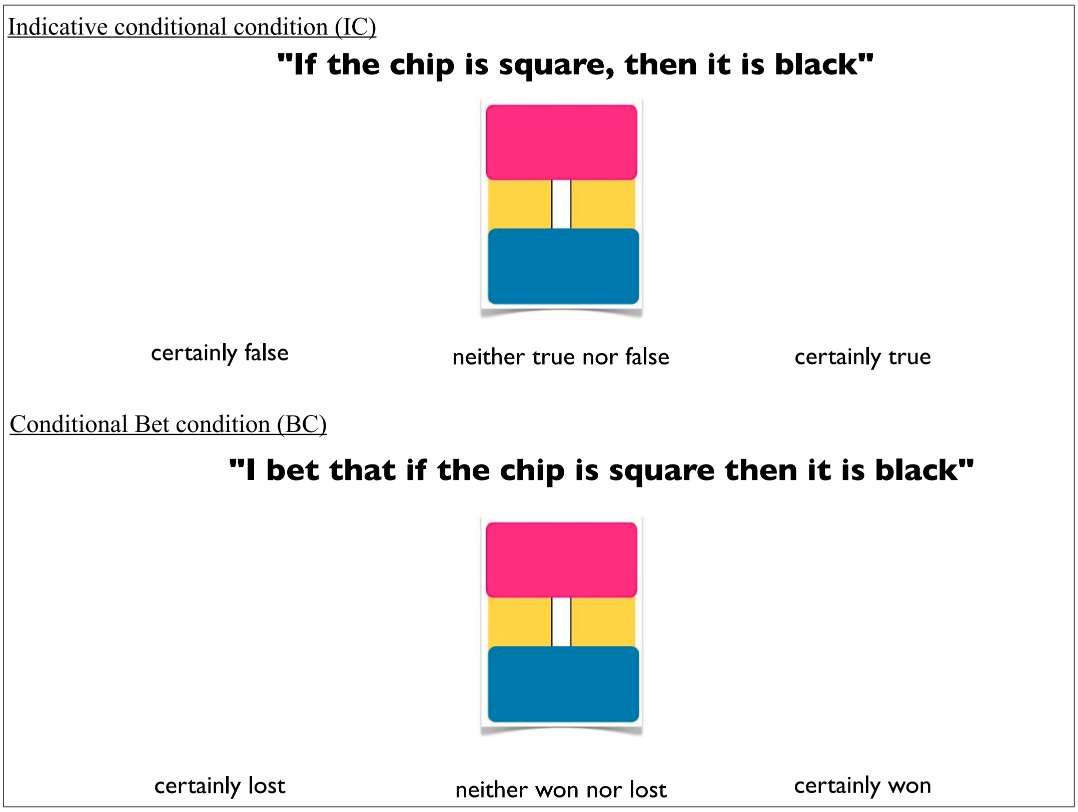

Table 1. The 2x2 de Finetti, “defective” table for *if A then C*: T = true, F = false, V = void.

		C	
		T	F
A	T	T	F
	F	V	V

Table 2. The 3x3 de Finetti table for *if A then C*: T = true, F = false, U = uncertain.

		C		
		T	U	F
	T	T	U	F
A	U	U	U	U
	F	U	U	U

Table 3. The cells of the 3x3 table.

		C		
		T	U	F
	T	*a*	*a/b*	*b*
A	U	*a/c*	*a/b/c/d*	*b/d*
	F	*c*	*c/d*	*d*

Table 4. The tables produced considering only two values for A and C - the first stage of our analysis - and the percentage of their occurrence (N = 192).

<table>
<tr><th></th><th>IC</th><th>BC</th><th>All</th></tr>
<tr><td>Conditional event
<table><tr><th>C|A</th><th>T</th><th>U</th><th>F</th></tr><tr><th>T</th><td>T</td><td></td><td>F</td></tr><tr><th>U</th><td></td><td></td><td></td></tr><tr><th>F</th><td>U</td><td></td><td>U</td></tr></table></td><td>38.6</td><td>45.1</td><td>41.7</td></tr>
<tr><td>Conjunction
<table><tr><th>A∧C</th><th>T</th><th>U</th><th>F</th></tr><tr><th>T</th><td>T</td><td></td><td>F</td></tr><tr><th>U</th><td></td><td></td><td></td></tr><tr><th>F</th><td>F</td><td></td><td>F</td></tr></table></td><td>35.6</td><td>34.1</td><td>34.9</td></tr>
<tr><td>Material conditional
<table><tr><th>A⊃C</th><th>T</th><th>U</th><th>F</th></tr><tr><th>T</th><td>T</td><td></td><td>F</td></tr><tr><th>U</th><td></td><td></td><td></td></tr><tr><th>F</th><td>T</td><td></td><td>T</td></tr></table></td><td>13.9</td><td>12.1</td><td>13</td></tr>
<tr><td>Material bi-conditional
<table><tr><th>A⇔C</th><th>T</th><th>U</th><th>F</th></tr><tr><th>T</th><td>T</td><td></td><td>F</td></tr><tr><th>U</th><td></td><td></td><td></td></tr><tr><th>F</th><td>F</td><td></td><td>T</td></tr></table></td><td>5.9</td><td>1.1</td><td>3.6</td></tr>
<tr><td>Bi-conditional
<table><tr><th>C||A</th><th>T</th><th>U</th><th>F</th></tr><tr><th>T</th><td>T</td><td></td><td>F</td></tr><tr><th>U</th><td></td><td></td><td></td></tr><tr><th>F</th><td>F</td><td></td><td>U</td></tr></table></td><td>1</td><td>4.4</td><td>2.6</td></tr>
<tr><td>Others</td><td>5</td><td>3.3</td><td>4.2</td></tr>
</table>

Table 5. Table answers (in %) in a three-valued framework (second stage analysis, N = 184*).

<table>
<tr><th>Conditions</th><th colspan="3">IC</th><th colspan="3">BC</th><th>All</th></tr>
<tr><th>Number of differences</th><th>0</th><th>1</th><th>Total</th><th>0</th><th>1</th><th>Total</th><th></th></tr>
<tr><td></td><td colspan="6">Conditional event</td><td>N = 80</td></tr>
<tr><td>de Finetti
<table><tr><th>$C|_{F}A$</th><th>T</th><th>U</th><th>F</th></tr><tr><th>T</th><td>T</td><td>U</td><td>F</td></tr><tr><th>U</th><td>U</td><td>U</td><td>U</td></tr><tr><th>F</th><td>U</td><td>U</td><td>U</td></tr></table></td><td>33.3</td><td>23.1</td><td>56.4</td><td>51.2</td><td>7.3</td><td>58.5</td><td>57.5</td></tr>
<tr><td>Farrell
<table><tr><th>$C|_{Fa}A$</th><th>T</th><th>U</th><th>F</th></tr><tr><th>T</th><td>T</td><td>U</td><td>F</td></tr><tr><th>U</th><td>U</td><td>U</td><td>F</td></tr><tr><th>F</th><td>U</td><td>U</td><td>U</td></tr></table></td><td>17.9</td><td>2.6</td><td>20.5</td><td>12.2</td><td>4.9</td><td>17.1</td><td>18.8</td></tr>
<tr><td>Cooper
<table><tr><th>$C|_{C}A$</th><th>T</th><th>U</th><th>F</th></tr><tr><th>T</th><td>T</td><td>U</td><td>F</td></tr><tr><th>U</th><td>T</td><td>U</td><td>F</td></tr><tr><th>F</th><td>U</td><td>U</td><td>U</td></tr></table></td><td>5.1</td><td>7.7</td><td>12.8</td><td>12.2</td><td>2.4</td><td>14.6</td><td>13.8</td></tr>
<tr><td>Ambiguous (Finetti-Cooper)
<table><tr><th>$C|_{a}A$</th><th>T</th><th>U</th><th>F</th></tr><tr><th>T</th><td>T</td><td>U</td><td>F</td></tr><tr><th>U</th><td>T</td><td>U</td><td>U</td></tr><tr><th>F</th><td>U</td><td>U</td><td>U</td></tr></table></td><td>10.3</td><td></td><td>10.3</td><td>9.8</td><td></td><td>9.8</td><td>10</td></tr>
<tr><td></td><td colspan="6">Conjunction</td><td>N = 67</td></tr>
<tr><td>de Finetti
<table><tr><th>$A\wedge_{F}C$</th><th>T</th><th>U</th><th>F</th></tr><tr><th>T</th><td>T</td><td>U</td><td>F</td></tr><tr><th>U</th><td>U</td><td>U</td><td>F</td></tr><tr><th>F</th><td>F</td><td>F</td><td>F</td></tr></table></td><td>47.2</td><td>5.6</td><td>52.8</td><td>41.9</td><td>9.7</td><td>51.6</td><td>52.2</td></tr>
<tr><td>Porte
<table><tr><th>$A\wedge_{P}C$</th><th>T</th><th>U</th><th>F</th></tr><tr><th>T</th><td>T</td><td>U</td><td>F</td></tr><tr><th>U</th><td>T</td><td>U</td><td>F</td></tr><tr><th>F</th><td>F</td><td>F</td><td>F</td></tr></table></td><td>19.4</td><td>8.3</td><td>27.8</td><td>25.8</td><td>3.2</td><td>29</td><td>28.4</td></tr>
<tr><td>Others</td><td></td><td></td><td>11.1</td><td></td><td></td><td>16.1</td><td>13.4</td></tr>
<tr><td>Ambiguous + indeterminate</td><td></td><td></td><td>8.3</td><td></td><td></td><td>3.2</td><td>6</td></tr>
<tr><td></td><td colspan="6">Material conditional</td><td>N = 25</td></tr>
<tr><td>de Finetti
<table><tr><th>$A\supset_{F}C$</th><th>T</th><th>U</th><th>F</th></tr><tr><th>T</th><td>T</td><td>U</td><td>F</td></tr><tr><th>U</th><td>T</td><td>U</td><td>U</td></tr><tr><th>F</th><td>T</td><td>T</td><td>T</td></tr></table></td><td>42.9</td><td></td><td>42.9</td><td>72.7</td><td></td><td>72.7</td><td>56</td></tr>
<tr><td>McCarthy
<table><tr><th>$A\supset_{Mc}C$</th><th>T</th><th>U</th><th>F</th></tr><tr><th>T</th><td>T</td><td>U</td><td>F</td></tr><tr><th>U</th><td>U</td><td>U</td><td>U</td></tr><tr><th>F</th><td>T</td><td>T</td><td>T</td></tr></table></td><td>35.7</td><td></td><td>35.7</td><td>9.1</td><td></td><td>9.1</td><td>24</td></tr>
<tr><td>Others</td><td></td><td></td><td>14.3</td><td></td><td></td><td>18.2</td><td>16</td></tr>
<tr><td>Ambiguous + indeterminate</td><td></td><td></td><td>7.1</td><td></td><td></td><td></td><td>4</td></tr>
</table>

Table 5. (continued)

Conditions	IC			BC			All
Number of differences	0	1	Total	0	1	Total	
	Material bi-conditional						N = 7
de Finetti	33.3	16.7	50		100	100	57.1

$A \Leftrightarrow_F C$	**T**	**U**	**F**
T	T	U	F
U	U	U	U
F	F	U	T

Conditions	IC 0	IC 1	IC Total	BC 0	BC 1	BC Total	All
Rescher***	16.7	16.7	33.3				28.6

$A \Leftrightarrow_R C$	**T**	**U**	**F**
T	T	U	F
U	U	U	F
F	F	F	T

Conditions	IC 0	IC 1	IC Total	BC 0	BC 1	BC Total	All
Ambiguous + indeterminate			16.7				14.3
	Bi-conditional						N = 5
de Finetti**	100		100		75		80

$C \vert\vert_F A$	**T**	**U**	**F**
T	T	U	F
U	U	U	U
F	F	U	U

Conditions	IC 0	IC 1	IC Total	BC 0	BC 1	BC Total	All
Farrell					25	25	20

$C \vert_{Fa} A$	**T**	**U**	**F**
T	T	U	F
U	U	U	F
F	F	F	U

* In this table we have removed the 8 subjects that are categorized as “others” in Table 4.
** de Finetti’s bi-conditional is not explicit in de Finetti (1936) but is easily deducible (see Baratgin et al. under review).
*** see Rescher’s T-split System (Rescher, 1969).